The Fourier Transform, Part I:

The Fourier transform is a mathematical method for describing a continuous function as a series of sine and cosine functions. The Fourier Transform is produced by applying a series of "Test Frequencies".

As an example, start with a signal acquired digitally as a series of N data points over a total time t_{signal}. This signal contains a single cosine wave with a frequency of ν Hz (ω rad/sec) and an amplitude of A.

Sample and signal parameters:

The signal frequency in Hz. $\nu_{signal} := (2 \cdot Hz)$

The signal frequency in radians per second. $\omega_{signal} := 2 \cdot \pi \cdot \nu_{signal}$

The signal amplitude $A := 1$

Number of data points $N := 512$

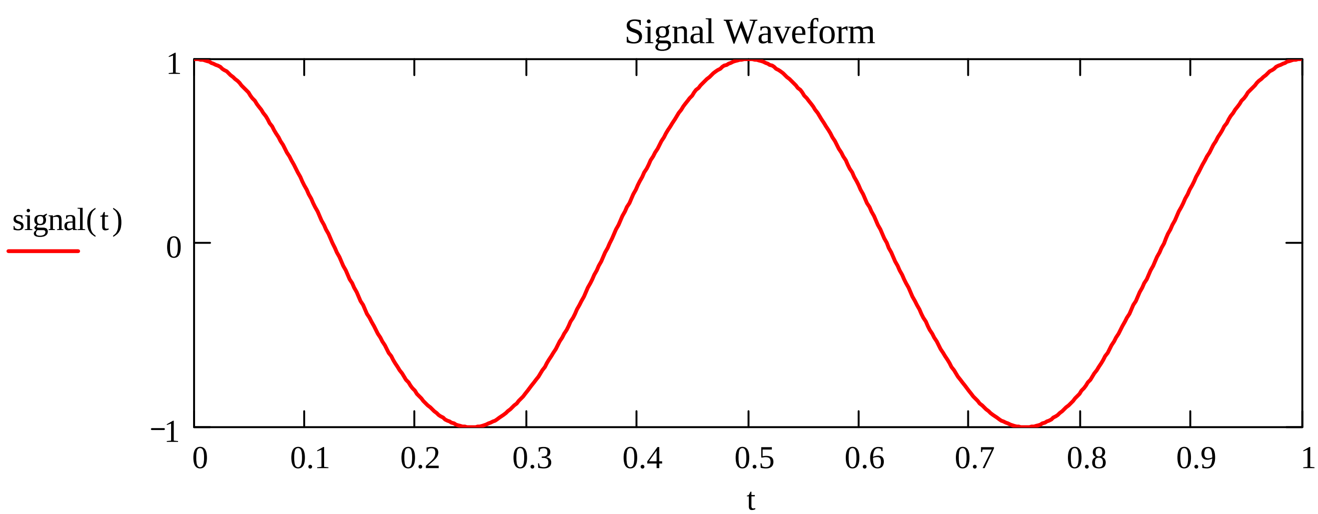
Total time the signal is acquired $t_{aquire} := 1 \cdot sec$

Indexes used for timing:

$$t := 0 \cdot sec, \frac{t_{aquire}}{N} .. \frac{(N-1) \cdot t_{aquire}}{N}$$

Calculate the signal waveform:

$$signal(t) := A \cdot \cos\left(\omega_{signal} \cdot t\right)$$



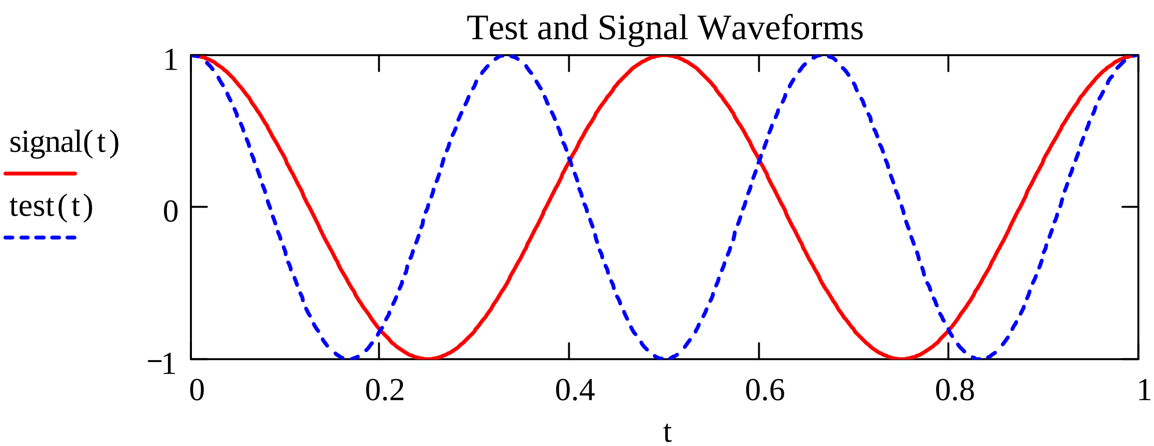
The Fourier transform determines the frequency components in the signal by applying a series of test frequencies. First the signal is multiplied by the test frequency to produce a new waveform. This new waveform is integrated to determine the amount of signal at the test frequency. Since the data is collected by the instrument as a series of digital points, the integration is performed numerically (by adding the discreet points). Alternatively, the function may be solved analytically with calculus.

Select a test frequency:

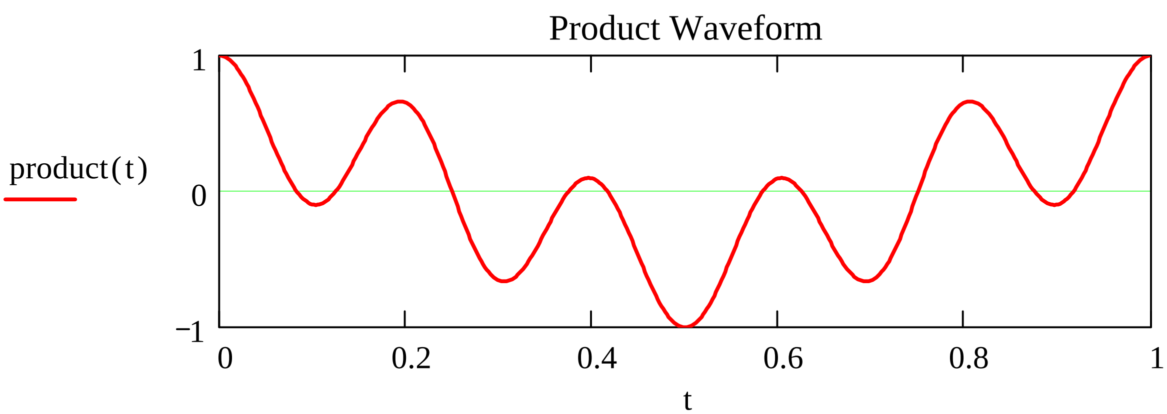
$\nu_{test} := 3 \cdot Hz$ $\qquad \omega_{test} := 2 \cdot \pi \cdot \nu_{test}$

Generate the test wave:

$\mathrm{test}(t) := \cos\left(\omega_{test} \cdot t\right)$



Multiply the two waveforms, point by point: $\mathrm{product}(t) := \mathrm{test}(t) \cdot \mathrm{signal}(t)$



Integrate the product function:

Analytically

$$\int_{0 \cdot sec}^{t_{aquire}} \mathrm{product}(t)\, dt = 0 \cdot sec$$

Numerically

$$\sum_{i=0}^{N-1} \frac{\mathrm{product}\left(i \cdot \frac{t_{aquire}}{N}\right)}{N} = 0$$

Questions.

Use this Mathcad document to answer the following questions. Play around with the variables (Highlighted in yellow) until you get a feeling for how the Fourier transform works.

1. Change the test frequency to 0, 1, 2, 3, 4, and 5 Hz and answer the following questions.

   a. Carefully look at the product waveform at each test frequency, is the integration zero?
   b. What is the numerical integration of the product waveform for each test frequency.
   c. What is the analytical integration of the product waveform at each test frequency.
   c. Graph these results (What x and y axies are appropriate for this graph?).

2. Set the signal amplitude to 10. Change the test frequency to 0, 1, 2, 3, 4, and 5 Hz and answer the following questions.
   a. Carefully look at the product waveform at each test frequency, is the integration zero?
   b. What is the numerical integration of the product waveform for each test frequency.
   c. What is the analytical integration of the product waveform at each test frequency.
   c. Graph these results (What x and y axies are appropriate for this graph?).

3. Set the signal frequency to 4 Hz. Change the test frequency to 0, 1, 2, 3, 4, and 5 Hz and answer the following questions.
   a. Carefully look at the product waveform at each test frequency, is the integration zero?
   b. What is the numerical integration of the product waveform for each test frequency.
   c. What is the analytical integration of the product waveform at each test frequency.
   c. Graph these results (What x and y axies are appropriate for this graph?).

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