Alan Hovaness
(1911 – )

Alleluia and Fugue, Op. 40b
and
Prayer of Saint Gregory
Chapter 6 - Outline

- Standing waves
  - Basics
  - Frequencies and wavelengths
  - Longitudinal waves

- Complex Waves

- Timbre
Standing Wave

- Produced when incident and reflected waves interfere.

- Principle of superposition.
Standing Wave

- There is no apparent motion along the direction in which the two individual waves move.
Standing Waves

Each loop $= \lambda/2$
Wavelengths of Standing Waves in a Rope

\[ n \left( \frac{\lambda_n}{2} \right) = L \]

\[ \lambda_n = \frac{2L}{n} \]
Frequencies of Standing Waves in a Rope

Since

\[ f = \frac{v}{\lambda} \]  then

\[ f_n = n\left(\frac{v}{2L}\right) \]
Video I-4

Transverse standing waves
Harmonic Series

A series of frequencies in which all members are an integral multiple of the lowest frequency
Harmonic Series

- The lowest frequency is called the **fundamental frequency** or **first harmonic**.
Harmonic Series

- The higher frequencies are called the
  second harmonic, third harmonic, fourth harmonic, etc.
Harmonic Series

\[ f_2 = 2f_1 \]
\[ f_3 = 3f_1 \]
\[ f_4 = 4f_1 \]

etc.
Harmonic Series

- Harmonics above the first are also called overtones.
Standing Sound Waves

Tube Open at Both Ends

\[ \lambda_1 = 2L \]
\[ f_1 = \frac{v}{\lambda_1} = \frac{v}{2L} \quad \text{First harmonic} \]

\[ \lambda_2 = L \]
\[ f_2 = \frac{v}{L} = 2f_1 \quad \text{Second harmonic} \]

\[ \lambda_3 = \frac{2}{3}L \]
\[ f_3 = \frac{3v}{2L} = 3f_1 \quad \text{Third harmonic} \]
Standing Sound Waves

*Tube Open at Both Ends:*

\[ \lambda_n = \frac{2L}{n} \]

\[ f_n = nf_1 \]
Standing Sound Waves

Tube Closed at Both Ends
(same as fixed rope)
Standing Sound Waves

Tube Closed at Both Ends:
(same as rope - fixed ends)

\[ \lambda_n = \frac{2L}{n} \]

\[ f_n = nf_1 \]
Standing Sound Waves

Tube Closed at One End:

\[
\begin{align*}
\lambda_1 &= 4L \\
f_1 &= \frac{v}{\lambda_1} = \frac{v}{4L} & \text{First harmonic} \\
\lambda_3 &= \frac{4}{3}L \\
f_3 &= \frac{3v}{4L} = 3f_1 & \text{Third harmonic} \\
\lambda_5 &= \frac{4}{5}L \\
f_5 &= \frac{5v}{4L} = 5f_1 & \text{Fifth harmonic}
\end{align*}
\]
Standing Sound Waves

**Tube Closed at One End:**

\[ \lambda_n = \frac{4L}{(2n-1)} \]

\[ f_n = (2n-1) \left( \frac{v}{4L} \right) \]
Standing Sound Waves

**Tube Closed at One End:**

\[
f_2 = 3 \ f_1
\]
\[
f_3 = 5 \ f_1
\]
\[
f_4 = 7 \ f_1
\]
Video III-1 and 2

Resonance in tubes
Complex Waves

Created when frequencies which are members of a harmonic series are added.
Complex Waves

Fundamental = f₁
Complex Waves

Same Amplitude

\[ f_1 + 2f_1 \]
Complex Waves

Same Amplitude

\[ f_1 + 2f_1 + 3f_1 \]
Complex Waves

Same Amplitude

\[ f_1 + 2f_1 + 3f_1 + 4f_1 \]
Complex Waves

Same Amplitude

$2f_1 + 3f_1 + 4f_1$ (fundamental removed)
Complex Waves

When waves whose frequencies are members of a harmonic series are added, the frequency of the resultant wave is always the same as that of the fundamental.
Auditory Demo

Missing Fundamental
(virtual pitch)
Track 37.

Virtual Pitch with Random
Harmonics
Track 43-45
Any periodic wave of frequency $f_1$ can be produced by adding together sine waves of frequency $f_1, 2f_1, 3f_1, 4f_1, 5f_1, \text{ etc.}$
Fourier Synthesis
Different Amplitudes
Fourier Synthesis

Square Wave

\[= (1)\sin(f) + (1/3)\sin(3f) + (1/5)\sin(5f) + (1/7)\sin(7f) + \ldots\]
Fourier Synthesis

Square Wave

Two Terms
Fourier Synthesis

Square Wave

Three Terms
Fourier Synthesis

Square Wave

Four Terms
Fourier Synthesis

Square Wave

Five Terms
Fourier Synthesis

Square Wave

12 Terms
Fourier Synthesis

Demo with Fourier Synthesizer and Oscilloscope
Fourier Analysis

- Any periodic wave of frequency $f_1$, no matter how complex, can be broken down into sine waves of frequency $f_1, 2f_1, 3f_1, 4f_1, 5f_1, \text{ etc.}$.
Fourier Analysis

The set of sine waves that make up a complex wave are called the complex wave’s Fourier Components.
Fourier Spectrum or Harmonic Spectrum

- A listing of the amplitudes of each component in either tabular or graphical form
Fourier Spectrum or Harmonic Spectrum

![Graph showing frequency spectrum with peaks at $f_1$, $2f_1$, $3f_1$, and $4f_1$.]
Timbre

- The different combinations of harmonics gives different qualities or timbers to sounds.
Timbre

Flute (few harmonics)
Timbre

Oboe (many harmonics)
Timbre

Violin (intense harmonics)
Auditory Demo

The Effect of spectrum on timbre

Track 53
III-6 Vibrations on a Guitar String

III-7 Fourier Analysis and Synthesis
Any periodic wave of frequency $f_1$ can be produced by adding together sine waves of frequency $f_1$, $2f_1$, $3f_1$, $4f_1$, $5f_1$, etc.
Summary

Any periodic wave of frequency $f_1$, no matter how complex, can be broken down into sine waves of frequency $f_1, 2f_1, 3f_1, 4f_1, 5f_1, \text{ etc.}$
Summary

- The pitch we hear always corresponds to that of the fundamental frequency.