Gustav Holst
(1874-1934)

The Planets - Saturn, the Bringer of Old Age
Musical Intervals

- Refer to Table 10.3 in the text.
- Intervals are ratios of frequencies or lengths.
Musical Intervals

- **Octave** (natural interval)
  - String Length Ratio = $2/1 = 2.00$
    - or $\frac{1}{2} = 0.50$
  - Frequency Ratio also = $2.00$
    - or $\frac{1}{2} = 0.50$
Musical Intervals
Musical Intervals

- **Perfect Fifth**
  - Seven semitones
  - String Length Ratio = $3/2 = 1.50$
  - or $2/3 = 0.6667$
  - Frequency Ratio also = 1.50
  - or $2/3 = 0.6667$
Musical Intervals
Musical Intervals

- **Perfect Fourth**
  - Five semitones
  - String Length Ratio = $\frac{4}{3} = 1.33$
    - or $\frac{3}{4} = 0.750$
  - String Length Ratio = $1.333$
    - or $\frac{3}{4} = 0.750$
Musical Intervals
Pythagorean Scale

- Pythagoras (582-507 BC)
- Ratios for intervals: 1.000, 1.333, 1.500, 2.000 (unison, fourth, fifth, octave)
- Used these ratios to construct a mathematical scale.
Pythagorean Scale

- Used string lengths since frequencies were not known.
- How do you divide an octave (1.00 to 2.00) into 8 equal parts?
- Or in terms of frequencies an interval such as 220 – 440 Hz?
Pythagoras's Rule

- Multiply or divide an existing length (ratio) by 3/2 (=1.500), factor of fifths.
- If the result lies between 1 and 2, leave it as it is.
Pythagoras’s Rule

- If the answer is less than 1, double it (up an octave)
- If the answer is greater than 1, halve it (down an octave)
Pythagoras’s Rule

Step #1

- Start with $D_4 = 1.000$ (293.7 Hz)
- Multiply $D_4 = 1.00$ by 1.50 to get 1.5 (the fifth) which is $A_4$ (440 Hz).
Pythagoras’s Rule

Step #2

- Start with $D_4 = 1.000$ (293.7 Hz)
- Divide $D_4 = 1.00$ by 1.50 to get 0.666 and double to get 1.333 (the fourth) which is $G_4$ (392 Hz).
Pythagoras’s Rule
Step #3

- Start with $A_4 = 1.500$ (440 Hz)

- Multiply $A_4 = 1.50$ by 1.50 to get 2.250 and halve to get 1.125 (the major second) which is $E_4$ (229.6 Hz).
Start with $G_4 = 1.333$ (292 Hz)

Divide $G_4 = 1.333$ by 1.50 to get $0.888888$ and double to get $1.777$ (the minor seventh) which is $C_5$ (523.3 Hz).
These first 5 notes D, E, G, A, C, and D again constitute the 5-note Chinese scale called **pentatonic** (5 tones)

Greek scales had 7 notes called **septatonic**
- Start with $E_4 = 1.125$ (229.6 Hz)

- Multiply $E_4 = 1.125$ by 1.50 to get 1.6875 (the major sixth) which is $B_4$ (493.9 Hz).
Step #6

- Start with $C_5 = 1.777$ (523.3 Hz)
- Divide $C_5 = 1.777$ by 1.50 to get 1.1851 (the minor third) which is $F_4$ (349.2 Hz).
Step #7

- Start with $G_4 = 1.333$ (292 Hz)

- Multiply $G_4 = 1.333$ by 1.50 to get 2.00 (the octave) which is $D_5$ (587.3 Hz).
Musical Intervals

Two different ratios between adjacent notes; semitone and tone.
Dorian Mode

Tone = T and Semitone = s

D E F G A B C D
Scales based on the white keys of the piano

Since there are seven different named keys A, B, C, D, E, F, G, there are seven modes.
Ionian Mode or Major Scale

C D E F G A B C

T T s T T T T T s

C D E F G A B C
Aoelian Mode or Minor Scale

\[\text{No musical notes provided in the text.}\\
\text{The text refers to a musical mode or scale.}\\
\text{The musical notation is not visible in the text.}\\
\text{The text does not contain any specific musical intervals or notes.}\\
\text{The text is about Aoelian Mode or Minor Scale.}\\
\text{The text provides a description of the mode or scale.}\\
\text{The text does not provide a specific musical notation.}\\
\]
Major and Minor Scales

- Major: J.S. Bach “Well Tempered Clavier Book II” Prelude I in C major. (Track #1)

- Minor: Prelude IV in C# minor. (Track #7)
Seven Modes

- C = Ionian (major scale)
- D = Dorian
- E = Phrygian (Spanish or Oriental)
- F = Lydian (funny, comical)
- G = Mixolydian
- A = Aeolian (minor scale)
- B = Locrian (not used)
Phrygian Mode

(starts on E)

Vaughn Williams

Fantasia on a Theme by Thomas Tallis
Multiples of 1.500 generate the same 8 note scale that was found by musicians to be the “right” ones for a musical scale.
The fifth is a multiple of 1.500.
The fifth is the 3\textsuperscript{rd} harmonic. \((3h_1/2h_1 = 1.50)\)
Mathematical Basis for Scales

- Third harmonic of A is E.
- Third harmonic of E is B.
- Third harmonic of B is F#.
- of F# is C#, of C# is G#, of G# is D#, of D# is A#, of A# is F, of F is C, of C is G, of G is D, and of D is back to A.
- This is the entire chromatic scale!
The 3\textsuperscript{rd} harmonic is the lowest and strongest harmonic that is not an octave.

Stringed instruments have the 3\textsuperscript{rd} harmonic.

A scale based on 3\textsuperscript{rd} harmonics should be the most “natural” or pleasing.
7 Greek modes or Church modes use all of the white keys.

Comprise 7 combinations of T=tone and s=semitone sequences.

Using a particular mode requires the scale to start on one and only one note.

Need to place semitones anywhere.
Equal-Tempered Scale

- E-F is a semitone and so is B-C.
- Add 5 more (the black keys).
Equal-Tempered Chromatic Scale

- 12 equally spaced semitones
- $\sqrt{2} \approx 1.059463...$
Major Scales

C Major

T   T   s   T   T  T   s   T   T   s   T   T  T   s

C  D  E  F  G  A  B  C
Major Scales

F Major

\[ \text{T T s T T T T s} \]

\[ \text{F G A Bb C D E F} \]
Major Scales Quality

- Happy
- Strong
- Serene
Minor Scales

A Minor

T s T T T s T T

A B C D E F G A

Studio - Scales.sng
Minor Scales Quality

- Sad
- Erie
- Troubling
Diatonic vs. Chromatic

- Diatonic Scales - major and minor scales
- Diatonic notes - notes of a particular scale
- Chromatic notes - the other notes
Equal Tempered vs. Pythagorean

- **Pythagorean** - the white keys determined by the rule of 3/2.
- **Equal Tempered** - 12 evenly spaced intervals by the factor of \( \sqrt[12]{2} \approx 1.059463... \)
- They are similar but not the same!
<table>
<thead>
<tr>
<th>Note</th>
<th>Equal Tempered</th>
<th>Pythagorean</th>
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</thead>
<tbody>
<tr>
<td>C</td>
<td>261.6</td>
<td>260.7</td>
</tr>
<tr>
<td>D</td>
<td>293.7</td>
<td>293.3</td>
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<tr>
<td>E</td>
<td>329.6</td>
<td>330.0</td>
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<tr>
<td>F</td>
<td>349.2</td>
<td>347.7</td>
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<tr>
<td>G</td>
<td>392.0</td>
<td>391.1</td>
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<tr>
<td>A</td>
<td>440.0</td>
<td>440.0</td>
</tr>
<tr>
<td>B</td>
<td>493.9</td>
<td>495.0</td>
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</tbody>
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