The Fourier Transform, Part IV:

The Fourier transform is a mathematical method for describing any continuous function as a series of sine and cosine functions. This document shows how the decay and phase of a signal effects the observed lineshape.

**The Signal**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Frequency (Hz)</td>
<td>$\nu := 5\text{-Hz}$</td>
</tr>
<tr>
<td>Phase (degrees)</td>
<td>$\phi := 0\text{-deg}$</td>
</tr>
<tr>
<td>Signal amplitude</td>
<td>$A := 1$</td>
</tr>
<tr>
<td>Relaxation time constant</td>
<td>$T_1 := 5\text{-sec}$</td>
</tr>
</tbody>
</table>

**The Test**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Frequency</td>
<td>$\nu_{\text{test}} := 5\text{-Hz}$</td>
</tr>
<tr>
<td>Sampling parameters</td>
<td></td>
</tr>
<tr>
<td>Number of data points</td>
<td>$N := 1024$</td>
</tr>
<tr>
<td>Total time the signal is acquired</td>
<td>acquire := 50\text{-sec}</td>
</tr>
</tbody>
</table>

**Indexes used for timing:**

$$i := 0, 1.. N - 1 \quad t_i := i \cdot \frac{\text{acquire}}{N} \quad j := 0, 1.. \frac{N}{2} - 1 \quad \text{frequency}_j := \frac{j}{\text{acquire}}$$

**Calculate the signal waveform:**

$$\text{signal}_i := A \cdot \cos(t_i \cdot \omega + \phi) \cdot e^{-\frac{t_i}{T_1}}$$

![Signal waveform graph](image-url)
Determine the real and imaginary components of the signal by applying the test frequency:

**Test Wave**

\[
test_{real_i} = \cos(t_i \omega_{test})
\]

\[
test_{imag_i} = \sin(t_i \omega_{test})
\]

**Product Wave**

\[
real_i = test_{real_i} \cdot signal_i
\]

\[
imag_i = test_{imag_i} \cdot signal_i
\]

**Integrate**

\[
signal_{real} = \sum_{i=0}^{N-1} \frac{real_i}{0.5 \cdot N}
\]

\[
signal_{imag} = \sum_{i=0}^{N-1} \frac{imag_i}{0.5 \cdot N}
\]

**Integrated Signal**

\[
signal_{real} = 0.101
\]

\[
signal_{imag} = 1.798 \cdot 10^{-5}
\]

\[
F := \text{fft}(signal)
\]
Questions.

1. Change the signal phase from 0 degrees to 360 degrees in 30 degree steps. For each step, observe the following:
   a. The phase of the signal waveform. What does the phase mean for the plot of signal vs. time?
   b. The phase shift between the real and imaginary product waveforms.
   c. The integrated signal for the real and imaginary spectra.
   d. The real and imaginary frequency spectra.
   e. What is the relationship between the integrated signal and the frequency spectra?

2. Change the signal phase to -30, -60, and -90 degrees. How does this effect the signal?

3. Change the acquisition time. How does this effect the signal?

4. Change the relaxation time constant. How does this effect the signal?