The Fourier Transform, Part III:

The Fourier transform is a mathematical method to describe a continuous function as a series of sine and cosine functions. This document shows how a combination of real and imaginary spectra (cosine and sine) describe the frequency and phase of a signal.

In this document the data sets are treated as arrays instead of as functions.

(Note: by convention the cosine component is called the real and the sine component is called the imaginary.)

Sampling parameters:

Number of data points \( N := 512 \)

Total time the signal is acquired \( \text{acquire} := 1 \cdot \text{sec} \)

Indexes used for timing:

\[ i := 0, 1, \ldots, N - 1 \]
\[ t_i := \frac{i}{N} \cdot \text{acquire} \]
\[ j := 0, 1, \ldots, \frac{N}{2} - 1 \]
\[ \text{frequency}_j := \frac{j}{\text{acquire}} \]

Calculations:

\[ \omega := 2 \cdot \pi \cdot v \]
\[ \omega_{\text{test}} := 2 \cdot \pi \cdot v_{\text{test}} \]

Signal

\[ \text{signal}_i := A \cdot \cos \left( t_i \cdot \omega + \phi \right) \]

Real

Test Wave

\[ \text{test real}_i := \cos \left( t_i \cdot \omega_{\text{test}} \right) \]

Product Wave

\[ \text{real}_i := \text{test real}_i \cdot \text{signal}_i \]

Imaginary

\[ \text{test imag}_i := \sin \left( t_i \cdot \omega_{\text{test}} \right) \]

Integrate

\[ \text{signal real} := \sum_{i = 0}^{N - 1} \frac{\text{real}_i}{0.5 \cdot N} \]

\[ \text{signal imag} := \sum_{i = 0}^{N - 1} \frac{\text{imag}_i}{0.5 \cdot N} \]

\[ F := \text{fft} \left( \text{signal} \right) \]
Signal

Product Waveforms

Frequency Spectrum

<table>
<thead>
<tr>
<th>Signal</th>
<th>Test</th>
<th>Integration</th>
</tr>
</thead>
<tbody>
<tr>
<td>ν = 5 Hz</td>
<td>ν_{test} = 5 Hz</td>
<td>signal_{real} = 0.5, signal_{imag} = -0.866</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Frequency</th>
<th>Phase</th>
<th>Amplitude</th>
</tr>
</thead>
<tbody>
<tr>
<td>Signal</td>
<td>ϕ = 60 deg</td>
<td>A = 1</td>
</tr>
<tr>
<td>Test</td>
<td></td>
<td></td>
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</tbody>
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Questions.

1. Change the signal phase from 0 degrees to 360 degrees in 30 degree steps. For each step, observe the following:
   a. The phase of the signal waveform. What does the phase mean for the plot of signal vs. time?
   b. The phase shift between the real and imaginary product waveforms.
   c. The integrated signal for the real and imaginary spectra.
   d. The real and imaginary frequency spectra.

2. Change the signal phase to -30, -60, and -90 degrees. How does this effect the signal?