The Fourier Transform, Part II:

The Fourier transform is a mathematical method for describing any continuous function as a series of sine and cosine functions. This document shows how the Fourier Transform extracts information from a signal with more than 1 frequency component.

The Signal

<table>
<thead>
<tr>
<th>Signal 1</th>
<th>Signal 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Frequency (rad/sec):</td>
<td>$\omega_1 := 2\pi \nu_1$</td>
</tr>
</tbody>
</table>

Sampling parameters:

- Number of data points: $N := 512$
- Total time the signal is acquired: $t_{\text{acquire}} := 1\text{-sec}$

Calculate the signal waveform:

- Signal 1: $\text{signal}_1(t) := A_1 \cos(t \omega_1)$
- Signal 2: $\text{signal}_2(t) := A_2 \cos(t \omega_2)$
- Sum of Signal 1 and 2: $\text{signal}(t) := \text{signal}_1(t) + \text{signal}_2(t)$

Graph the waveforms:
Now determine the frequency components in the signal using the Fourier Transform:

Generate the test wave:

Test frequency (Hz): \( \nu_{\text{test}} = 10 \cdot \text{Hz} \)

Test Wave

\[
\text{test}(t) := \cos\left( t \cdot \omega_{\text{test}} \right)
\]

Multiply the two waveforms, and integrate to perform FT:

\[
\text{product}(t) := \text{test}(t) \cdot \text{signal}(t)
\]

The Settings (Change Here):

<table>
<thead>
<tr>
<th>Signal 1</th>
<th>Signal 2</th>
<th>Test</th>
</tr>
</thead>
<tbody>
<tr>
<td>Frequency</td>
<td>( \nu_1 = 10 \cdot \text{Hz} )</td>
<td>( \nu_2 = 3 \cdot \text{Hz} )</td>
</tr>
<tr>
<td>Amplitude</td>
<td>( A_1 = 2 )</td>
<td>( A_2 = 1 )</td>
</tr>
</tbody>
</table>

Integrated area of product waveform

\[
\int_{0 \cdot \text{sec}}^{t_{\text{acquire}}} \text{product}(t) \, dt = 1 \cdot \text{sec}
\]
Questions

1. Set the Signal Frequencies to 3 and 10 Hz Change the Test frequency and record the Integrated area of the product waveform. Try test frequencies of 1, 2, 3 ... 15 Hz. Graph your results. What is the significance of these results?

2. Set Signal 1 and the Test frequency to 5 Hz. Change the Signal 2 frequency. Does this have any effect on the Integrated area? Why?

3. Change the amplitude of the Signal 1 and Signal 2. What effect does this have on the integrated area? Why?

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