



The Planets - Saturn, the
Bringer of Old Age



- ***Refer to Table 10.3 in the text.***
- ***Intervals are ratios of frequencies or lengths.***



● ***Octave*** (natural interval)

◆ String Length Ratio = $2/1 = 2.00$

or $1/2 = 0.50$

◆ Frequency Ratio also = 2.00

or $1/2 = 0.50$

A musical staff with five red lines. On the left, three yellow notes ascend from the first space to the second line. On the right, two yellow notes descend from the second line to the first space. The text "Musical Intervals" is written in yellow across the middle of the staff.

Musical Intervals





● *Perfect Fifth*

- ◆ Seven semitones
- ◆ String Length Ratio = $3/2 = 1.50$
or $2/3 = 0.6667$
- ◆ Frequency Ratio also = 1.50
or $2/3 = 0.6667$

A musical staff with five red lines. On the left, three yellow notes ascend from the first space to the second line. On the right, two yellow notes descend from the second line to the first space. The text "Musical Intervals" is written in yellow across the middle of the staff.

Musical Intervals





● *Perfect Fourth*

- ◆ Five semitones
- ◆ String Length Ratio = $4/3 = 1.33$
or $3/4 = 0.750$
- ◆ String Length Ratio = 1.333
or $3/4 = 0.750$

A musical staff with five red lines. On the left, three yellow notes ascend from the first space to the second line. On the right, two yellow notes descend from the second space to the first space. The text "Musical Intervals" is written in yellow across the middle of the staff.

Musical Intervals



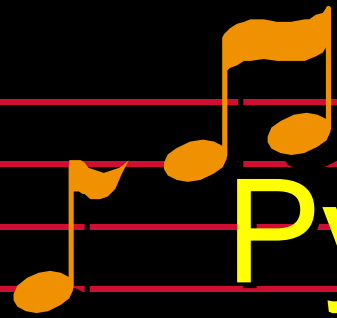


- Pythagoras (582-507 BC)
- Ratios for intervals: 1.000, 1.333, 1.500, 2.000 (unison, fourth, fifth, octave)
- Used these ratios to construct a mathematical scale.

A musical staff with five red lines and a red border. The staff contains several yellow musical notes of varying heights and positions. The text 'Pythagorean Scale' is written in yellow across the middle of the staff.

Pythagorean Scale

- Used string lengths since frequencies were not known.
- How do you divide an octave (1.00 to 2.00) into 8 equal parts?
- Or in terms of frequencies an interval such as 220 –440 Hz?



Pythagoras's Rule



- Multiply or divide an existing length (ratio) by $\frac{3}{2}$ (=1.500), factor of fifths.
- If the result lies between 1 and 2, leave it as it is.



Pythagoras's Rule

- If the answer is less than 1, double it (up an octave)
- If the answer is greater than 1, halve it (down an octave)

A musical staff with five red lines and a red border. It contains several yellow musical notes: a quarter note on the first line, a quarter note on the second line, a quarter note on the third line, a quarter note on the fourth line, and a quarter note on the fifth line. The text "Pythagoras's Rule" is written in yellow across the staff, and "Step #1" is written in yellow below the staff.

Pythagoras's Rule

Step #1

- Start with $D_4 = 1.000$
(293.7 Hz)
- Multiply $D_4 = 1.00$ by 1.50
to get 1.5 (the fifth) which
is A_4 (440 Hz).

A musical staff with five red lines and a red C-clef on the first line. The staff contains several yellow musical notes: a quarter note on the first line, a quarter note on the second line, a quarter note on the second space, a quarter note on the third line, and a quarter note on the third space. The text "Pythagoras's Rule" is written in yellow across the staff, and "Step #2" is written in yellow below the staff.

Pythagoras's Rule

Step #2

- Start with $D_4 = 1.000$ (293.7 Hz)
- Divide $D_4 = 1.00$ by 1.50 to get 0.666 and double to get 1.333 (the fourth) which is G_4 (392 Hz).

A musical staff with five red lines and a red C-clef on the first line. The staff contains several yellow musical notes: a quarter note on the first line, a quarter note on the second line, a quarter note on the second space, and a quarter note on the third line. The text "Pythagoras's Rule" is written in yellow across the staff, and "Step #3" is written in yellow below the staff.

Pythagoras's Rule

Step #3

- Start with $A_4 = 1.500$ (440 Hz)
- Multiply $A_4 = 1.50$ by 1.50 to get 2.250 and halve to get 1.125 (the major second) which is E_4 (229.6 Hz).

A musical staff with five red lines and a red border. It contains several yellow musical notes: a quarter note on the first line, a quarter note on the second line, a quarter note on the third space, a quarter note on the fourth space, and a quarter note on the fifth space. The text "Pythagoras's Rule" is written in yellow across the staff, and "Step #4" is written in yellow below the staff.

Pythagoras's Rule

Step #4

- Start with $G_4 = 1.333$ (292 Hz)
- Divide $G_4 = 1.333$ by 1.50 to get 0.88888 and double to get 1.777 (the minor seventh) which is C_5 (523.3 Hz).



- These first 5 notes D, E, G, A, C, and D again constitute the 5-note Chinese scale called **pentatonic** (5 tones)
- Greek scales had 7 notes called **septatonic**

A musical staff with five red lines and a red border. It contains several yellow musical notes of varying heights and stems. The text 'Pythagoras's Rule' is written in yellow across the staff, and 'Step #5' is written below it in yellow.

Pythagoras's Rule

Step #5

- Start with $E_4 = 1.125$ (229.6 Hz)
- Multiply $E_4 = 1.125$ by 1.50 to get 1.6875 (the major sixth) which is B_4 (493.9 Hz).

A musical staff with five red lines and a red border. It contains several yellow musical notes: a quarter note on the first line, a quarter note on the second line, a quarter note on the third line, a quarter note on the fourth line, and a quarter note on the fifth line. The text "Pythagoras's Rule" is written in yellow across the staff, and "Step #6" is written in yellow below it.

Pythagoras's Rule

Step #6

- Start with $C_5 = 1.777$ (523.3 Hz)
- Divide $C_5 = 1.777$ by 1.50 to get 1.1851 (the minor third) which is F_4 (349.2 Hz).

A musical staff with five red lines and a red border. It contains several yellow musical notes: a quarter note on the first line, a quarter note on the second line, a quarter note on the third line, a quarter note on the fourth line, and a quarter note on the fifth line. The text "Pythagoras's Rule" is written in yellow across the staff, and "Step #7" is written in yellow below the staff.

Pythagoras's Rule

Step #7

- Start with $G_4 = 1.333$ (292 Hz)
- Multiply $G_4 = 1.333$ by 1.50 to get 2.00 (the octave) which is D_5 (587.3 Hz).

A musical staff with five red lines. On the left, there are three yellow notes: a quarter note on the first line, an eighth note on the second line, and a quarter note on the second space. On the right, there are two yellow notes: a quarter note on the second space and a quarter note on the second line. The text "Musical Intervals" is written in yellow across the middle of the staff.

Musical Intervals



Two different ratios between adjacent notes; semitone and tone.



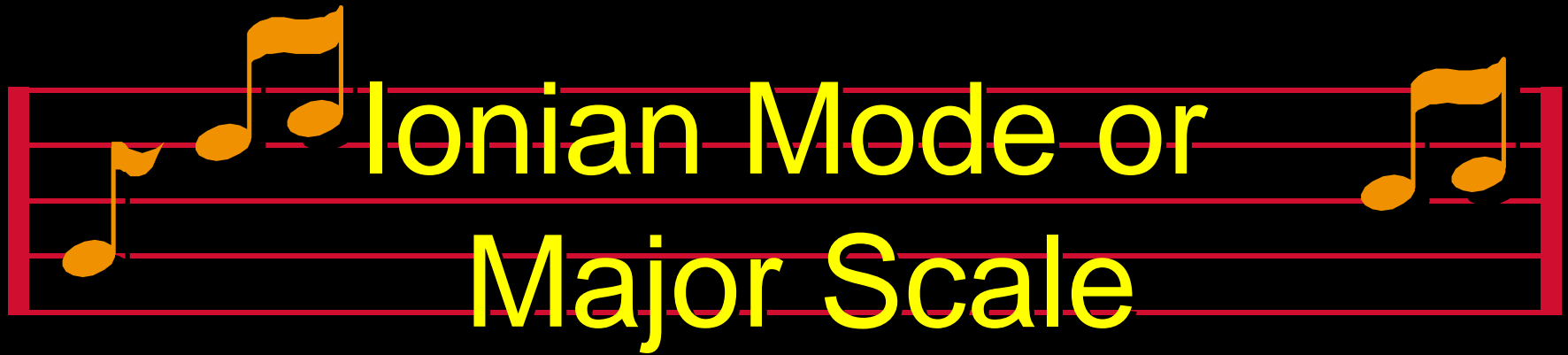
Tone = **T** and Semitone = **s**

T s T T T s T

D E F G A B C D



- Scales based on the white keys of the piano
- Since there are seven different named keys A, B, C, D, E, F, G, there are seven modes.



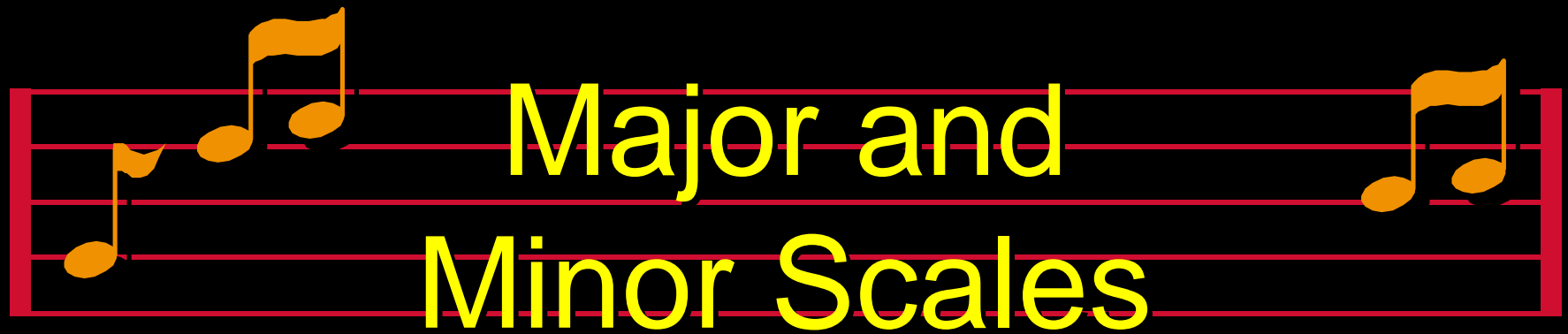
T T s T T T s

C D E F G A B C



T s T T s T T

A B C D E F G A



- Major: J.S. Bach “Well Tempered Clavier Book II” Prelude I in C major. (Track #1)
- Minor: Prelude IV in C# minor. (Track #7)



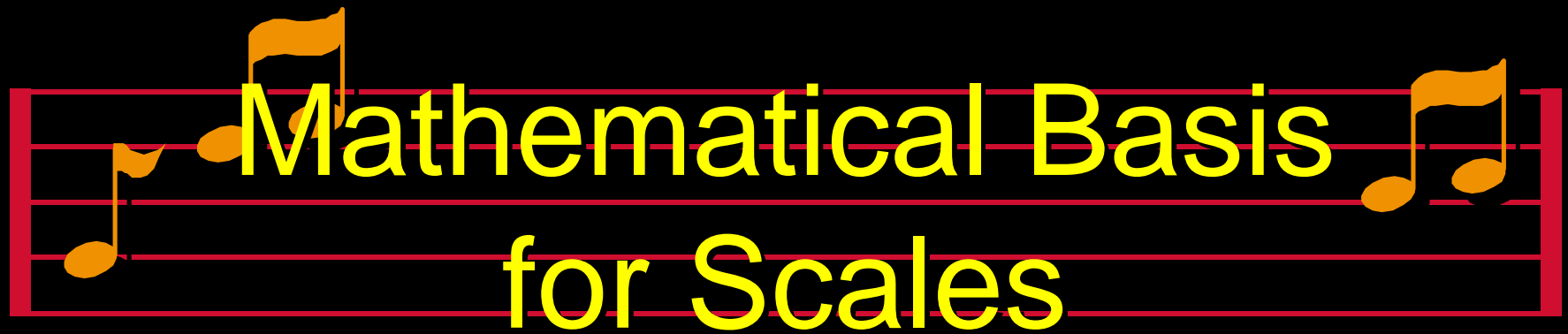
- C = **Ionian** (major scale)
- D = **Dorian**
- E = **Phrygian** (Spanish or Oriental)
- F = **Lydian** (funny, comical)
- G = **Mixolydian**
- A = **Aeolian** (minor scale)
- B = **Locrian** (not used)



(starts on E)

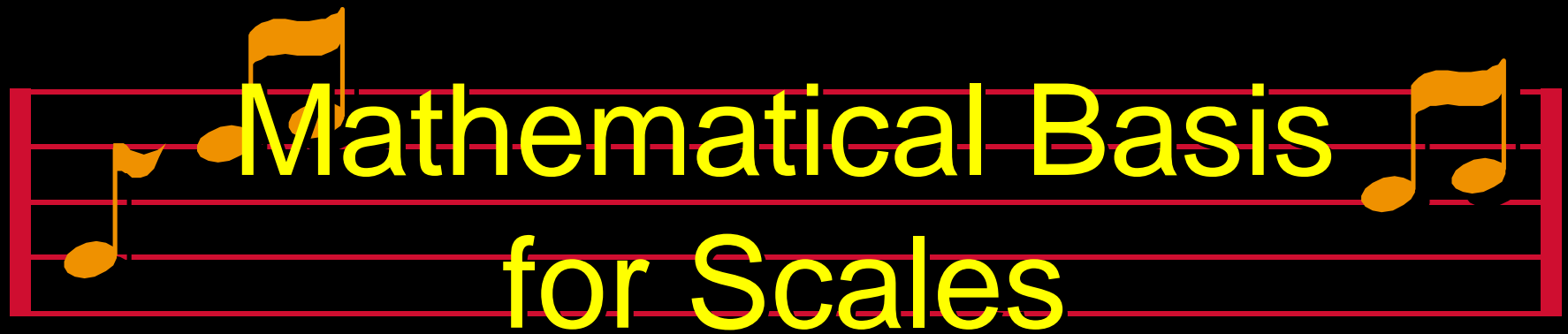
Vaughn Williams

Fantasia on a Theme by
Thomas Tallis

A graphic of a musical staff with five red lines and a red border. It contains several yellow musical notes of varying heights and positions. The text "Mathematical Basis for Scales" is written in yellow across the staff.

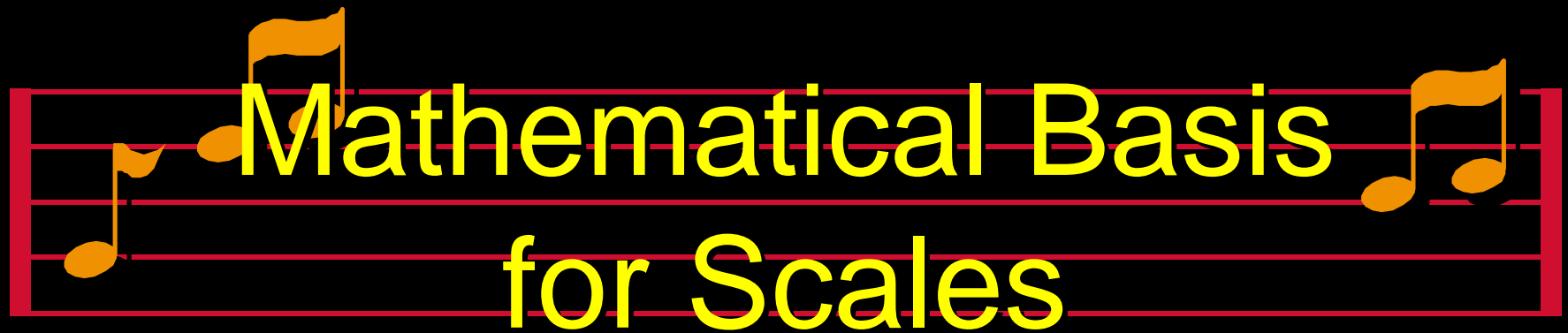
Mathematical Basis for Scales

Multiples of 1.500
generate the same 8 note
scale that was found by
musicians to be the “right”
ones for a musical scale.

A musical staff with five red lines and a red border. It contains four yellow musical notes: a quarter note on the first line, a quarter note on the second line, a quarter note on the third line, and a quarter note on the fourth line. The text "Mathematical Basis for Scales" is written in yellow across the staff.

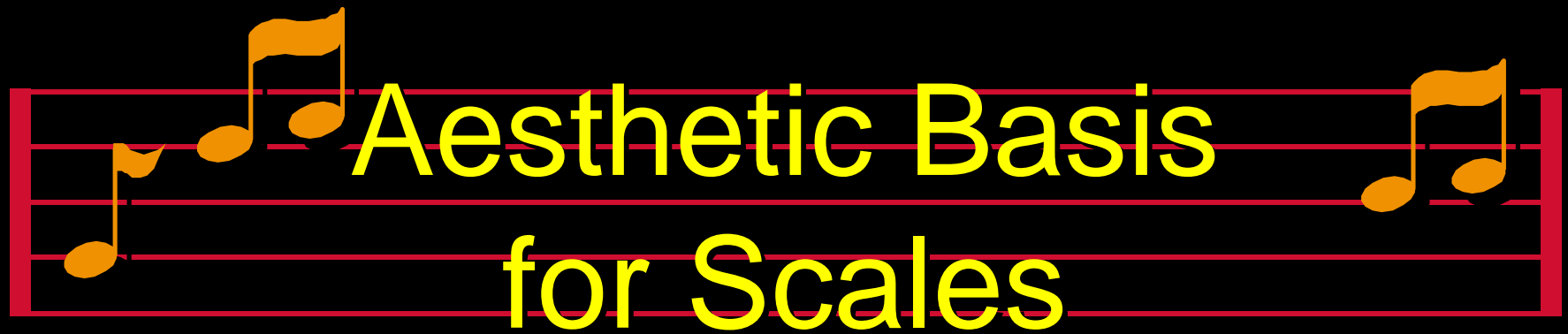
Mathematical Basis for Scales

- The fifth is a multiple of 1.500.
- The fifth is the 3rd harmonic. ($3h_1/2h_1 = 1.50$)

A musical staff with five red lines and a red border. It contains several yellow musical notes: a quarter note on the first line, a quarter note on the second line, a quarter note on the second space, a quarter note on the third line, a quarter note on the third space, a quarter note on the fourth line, and a quarter note on the fourth space. The text "Mathematical Basis for Scales" is written in yellow across the staff.

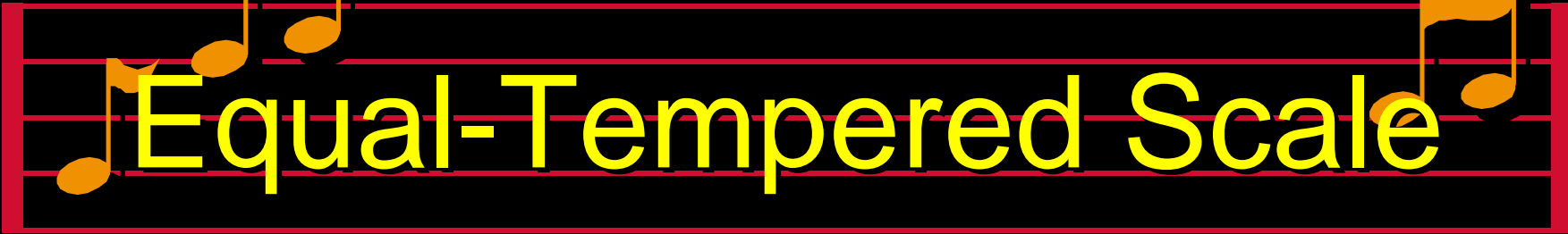
Mathematical Basis for Scales

- Third harmonic of A is E.
- Third harmonic of E is B.
- Third harmonic of B is F#.
- of F# is C#, of C# is G#, of G# is D#, of D# is A#, of A# is F, of F is C, of C is G, of G is D, and of D is back to A.
- This is the entire chromatic scale!

A musical staff with five red lines and a red border. It contains several yellow musical notes of varying pitch and rhythm, including quarter and eighth notes with stems and flags. The notes are arranged in a sequence that suggests a scale or melody.

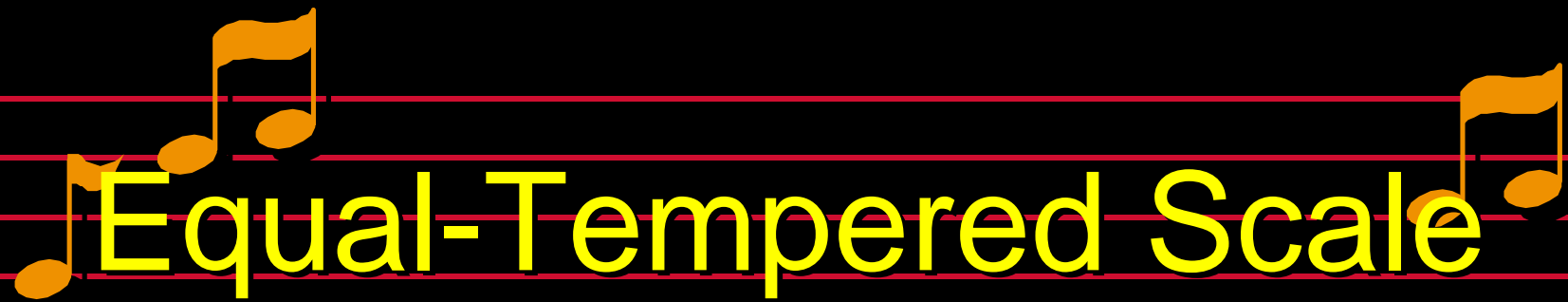
Aesthetic Basis for Scales

- The 3rd harmonic is the lowest and strongest harmonic that is not an octave.
- Stringed instruments have the 3rd harmonic.
- A scale based on 3rd harmonics should be the most “natural” or pleasing.

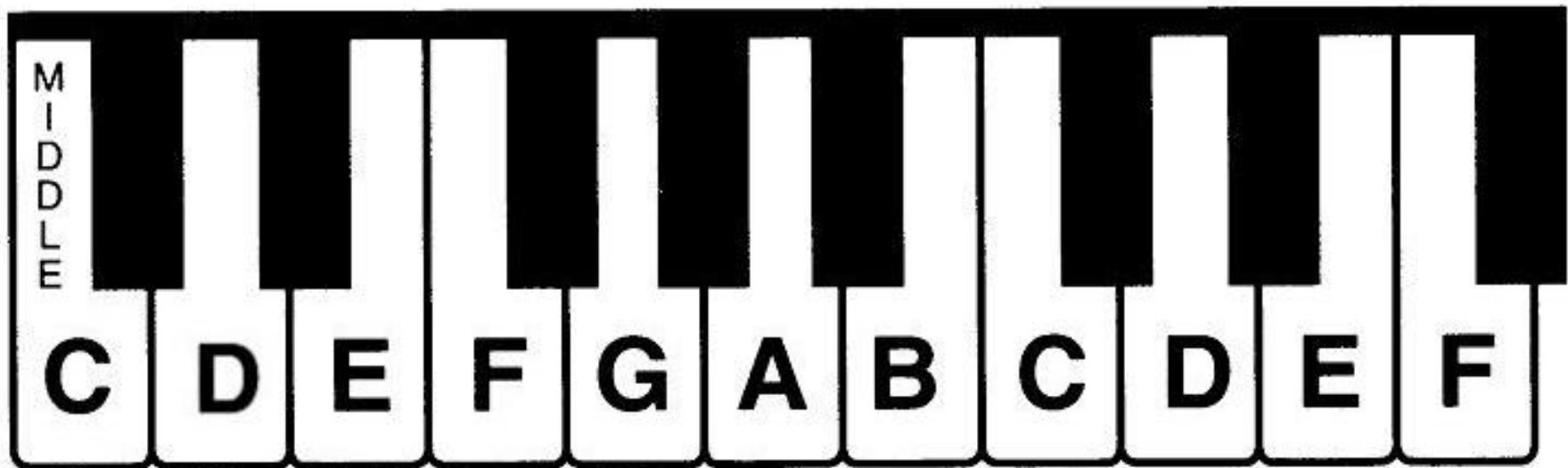
A musical staff with five red lines and a red border. The title 'Equal-Tempered Scale' is written in yellow text across the staff. There are four yellow musical notes: a quarter note on the first line, a quarter note on the second line, a quarter note on the third space, and a quarter note on the fourth space.

Equal-Tempered Scale

- 7 Greek modes or Church modes use all of the white keys
- Comprise 7 combinations of T=tone and s=semitone sequences
- Using a particular mode requires the scale to start on one and only one note.
- Need to place semitones anywhere.

A musical staff with five red lines and four yellow notes. The notes are positioned on the first, second, third, and fourth lines from the bottom. The word "Equal-Tempered Scale" is written in yellow across the staff.

Equal-Tempered Scale



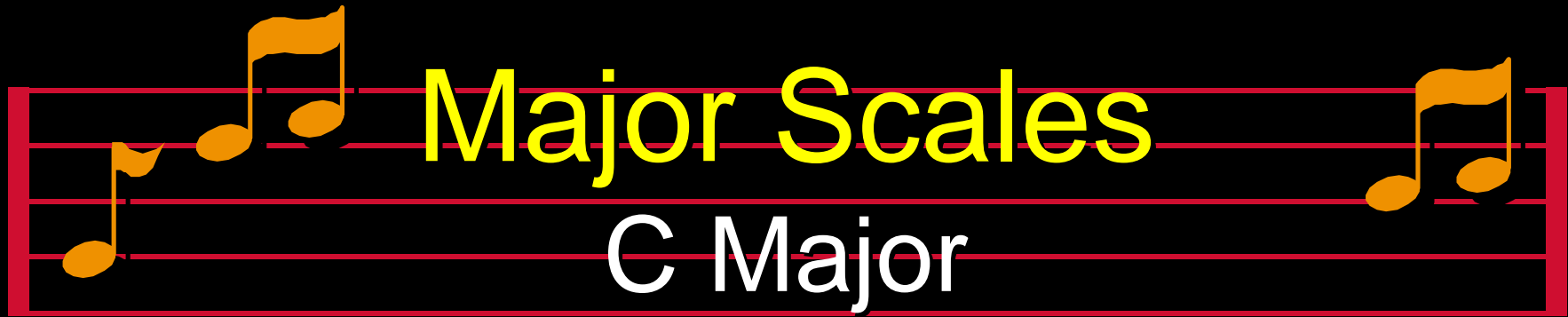
- E-F is a semitone and so is B-C.
- Add 5 more (the black keys).

Equal-Tempered Chromatic Scale



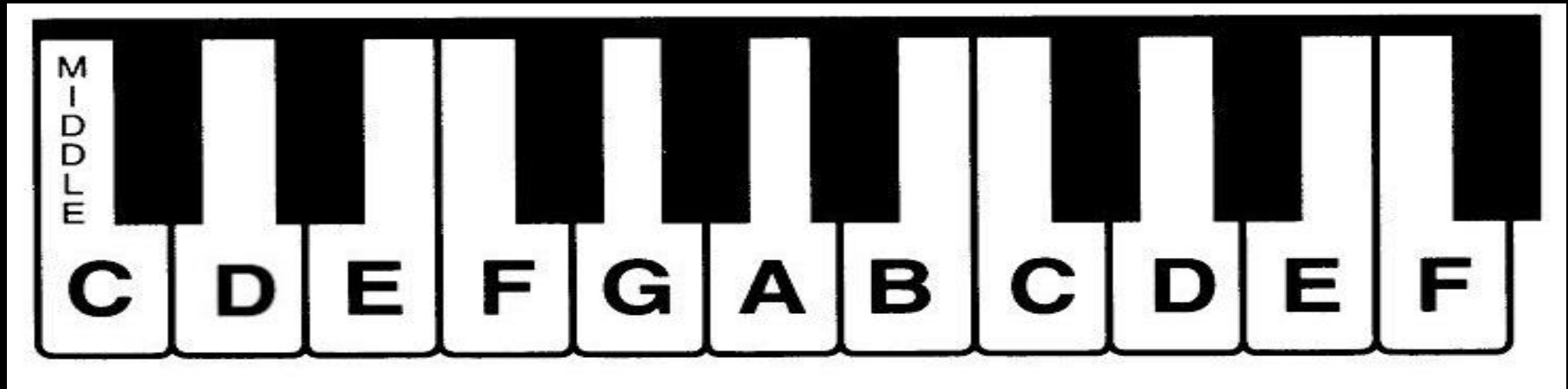
- 12 equally spaced semitones

- $\sqrt[12]{2} \approx 1.059463\dots$



Major Scales
C Major

A musical staff with five red lines. On the left, a sequence of five eighth notes (C, D, E, F, G) is shown. On the right, a sequence of two eighth notes (A, B) is shown. The title 'Major Scales' is written in yellow, and 'C Major' is written in white below it.



T T s T T T s
C D E F G A B C

A diagram showing the interval pattern for the C major scale. The first row contains the letters T, T, s, T, T, T, s in yellow. The second row contains the letters C, D, E, F, G, A, B, C in white.



F Major

T T s T T T s

F G A B_b C D E F

Studio - Scales.sng

A musical staff with five red lines and a red border. It contains several yellow musical notes of varying heights and positions. The text 'Major Scales Quality' is written in yellow across the staff.

Major Scales Quality

- Happy
- Strong
- Serene

Minor Scales



A Minor



T s T T s T T

A B C D E F G A



A Minor

T s T T s T T

A B C D E F G A

Studio - Scales.sng

A musical staff with five red lines and a red border. It contains several yellow musical notes: a quarter note on the first line, a quarter note on the second line, a quarter note on the second space, a quarter note on the third line, and a quarter note on the third space. The text "Minor Scales Quality" is written in yellow across the staff.

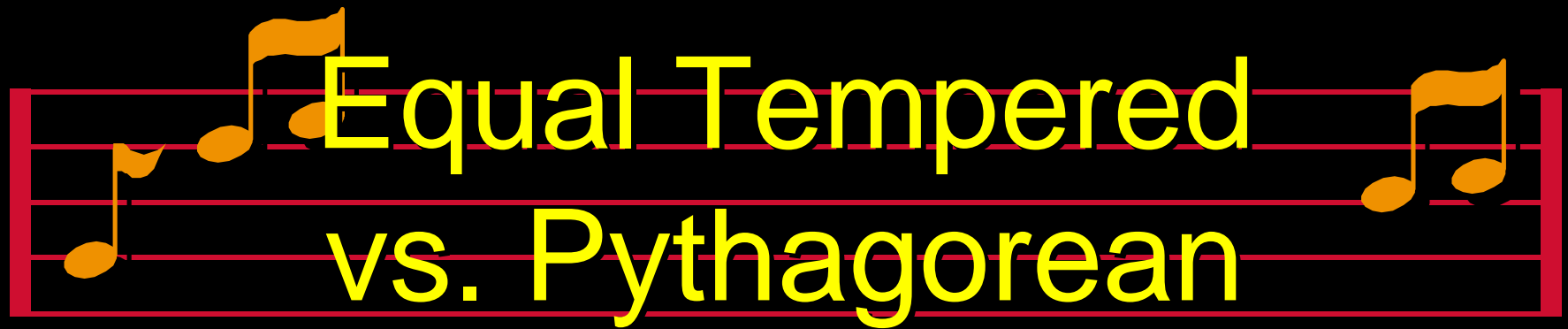
Minor Scales Quality

- Sad
- Erie
- Troubling

A musical staff with five red lines and a red border. It contains several yellow musical notes of varying heights and positions. The title "Diatonic vs. Chromatic" is written in yellow text across the staff.

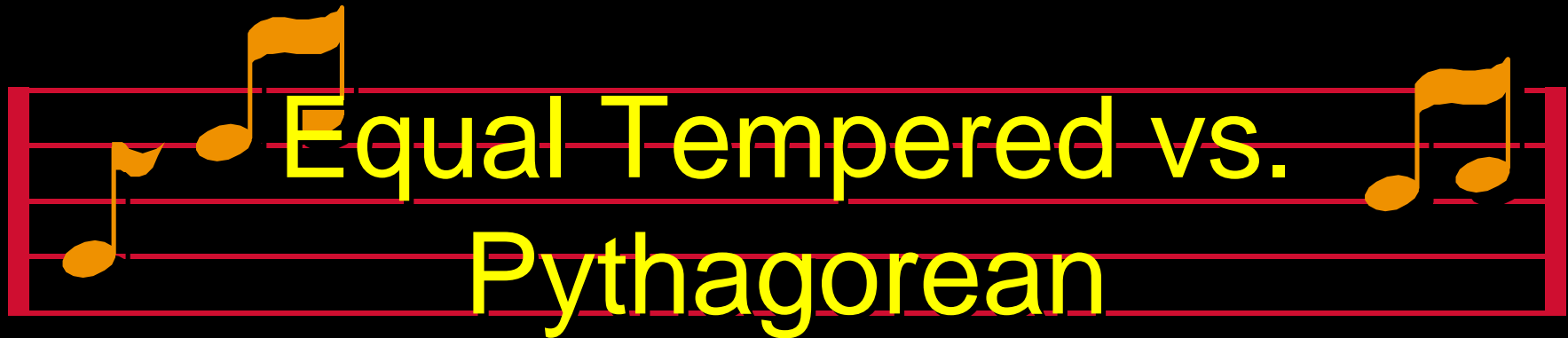
Diatonic vs. Chromatic

- **Diatonic Scales** - major and minor scales
- **Diatonic notes** - notes of a particular scale
- **Chromatic notes** - the other notes

A musical staff with five red lines and four orange notes. The notes are positioned on the first, second, and fourth lines from the bottom. The text 'Equal Tempered vs. Pythagorean' is written in yellow across the staff.

Equal Tempered vs. Pythagorean

- **Pythagorean** - the white keys determined by the rule of 3/2.
- **Equal Tempered** - 12 evenly spaced intervals by the factor of
$$\sqrt[12]{2} \approx 1.059463\dots$$
- They are similar but not the same!

A musical staff with five red lines and a red border. It contains three yellow notes: a quarter note on the first line, a quarter note on the second line, and a quarter note on the third line. The text "Equal Tempered vs. Pythagorean" is written in yellow across the staff.

Equal Tempered vs. Pythagorean

Note	Equal Tempered	Pythagorean
C	261.6	260.7
D	293.7	293.3
E	329.6	330.0
F	349.2	347.7
G	392.0	391.1
A	440.0	440.0
B	493.9	495.0