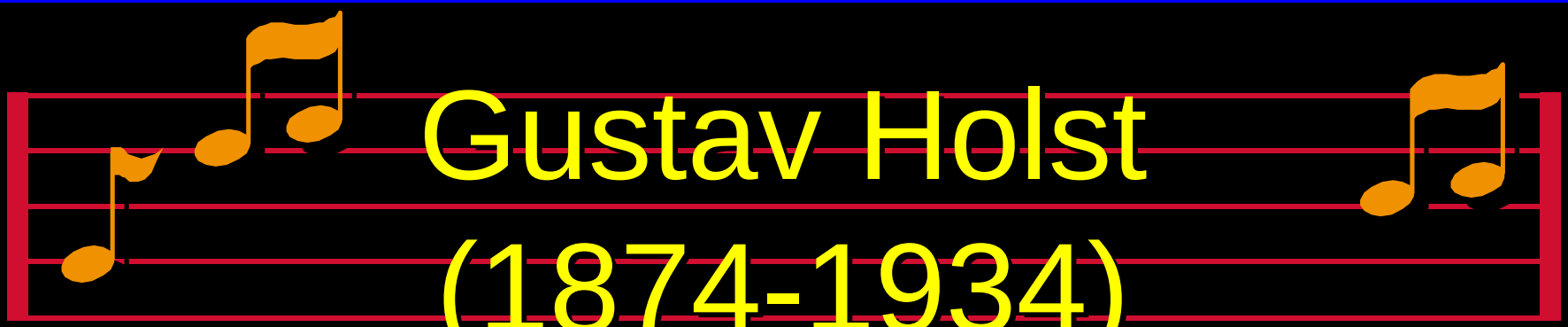
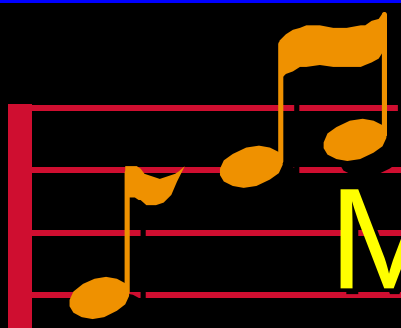


Gustav Holst (1874-1934)

The Planets - Saturn, the Bringer of Old Age



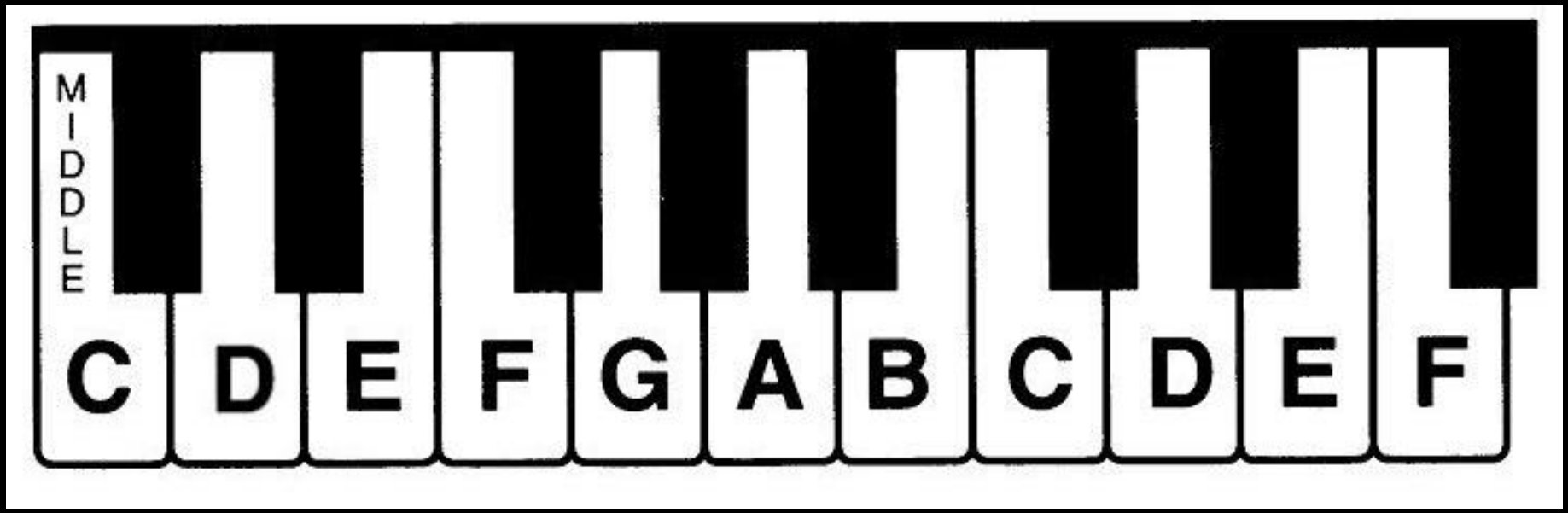
Musical Intervals

- ***Refer to Table 10.3 in the text.***
- ***Intervals are ratios of frequencies or lengths.***

Musical Intervals

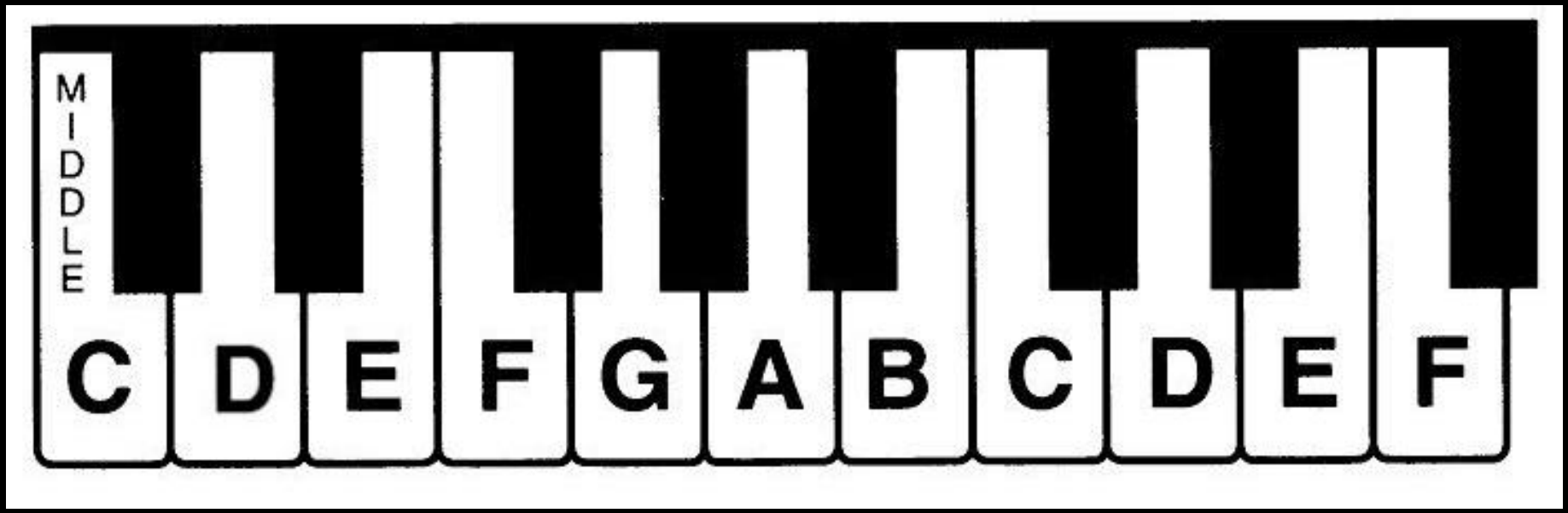
- ***Octave*** (natural interval)
  - String Length Ratio = 2/1 = 2.00
    or ½ = 0.50
  - Frequency Ratio also = 2.00
    or ½ = 0.50

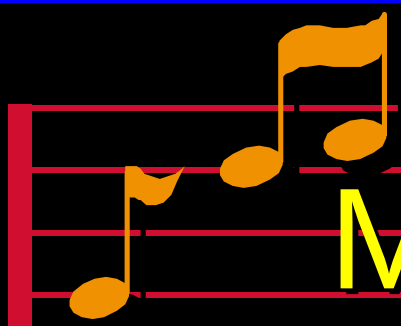
Musical Intervals



- ***Perfect Fifth***
  - Seven semitones
  - String Length Ratio = 3/2 =1.50 or 2/3 = 0.6667
  - Frequency Ratio also = 1.50 or 2/3 = 0.6667

Musical Intervals



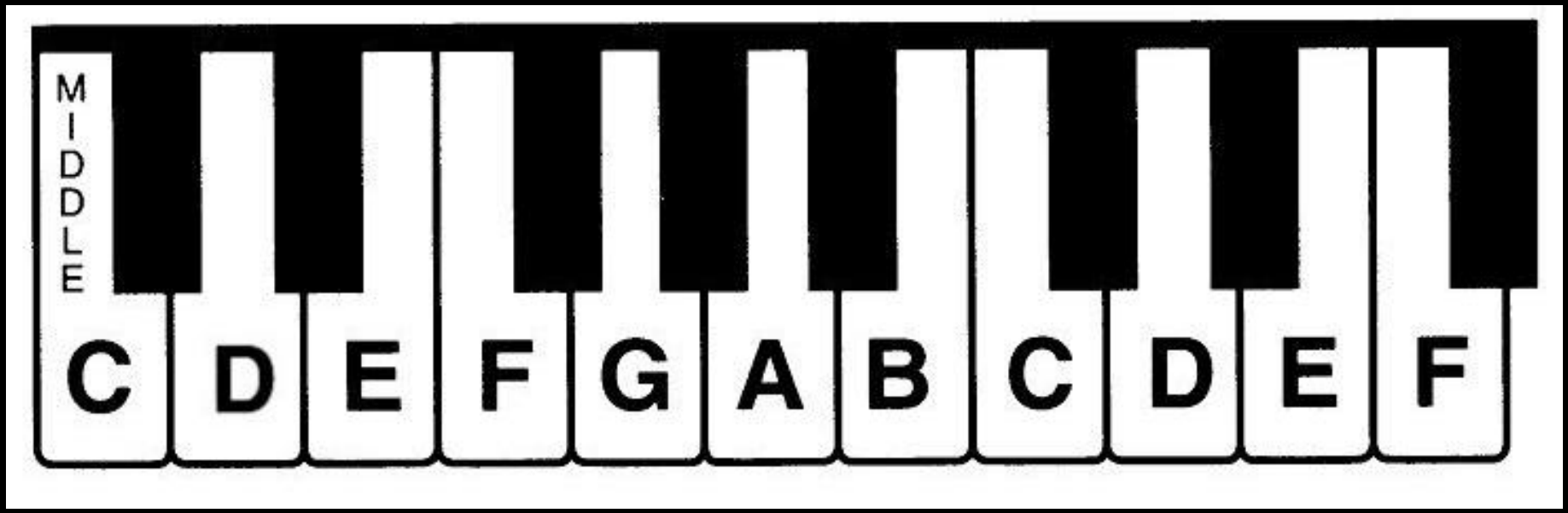


Musical Intervals

Perfect Fourth

- Five semitones
- String Length Ratio = 4/3 = 1.33
  or ¾ = 0.750
- String Length Ratio = 1.333
  or ¾ = 0.750

Musical Intervals

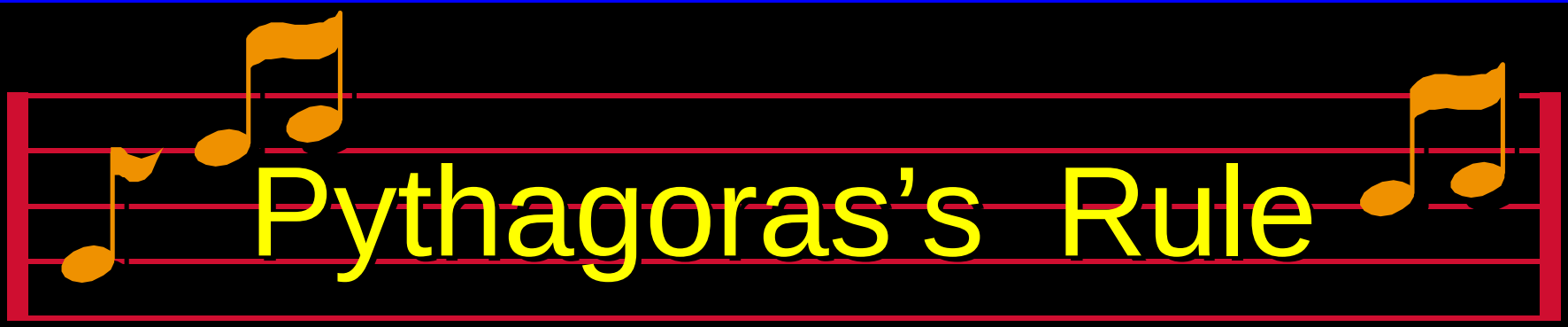


Pythagorean Scale

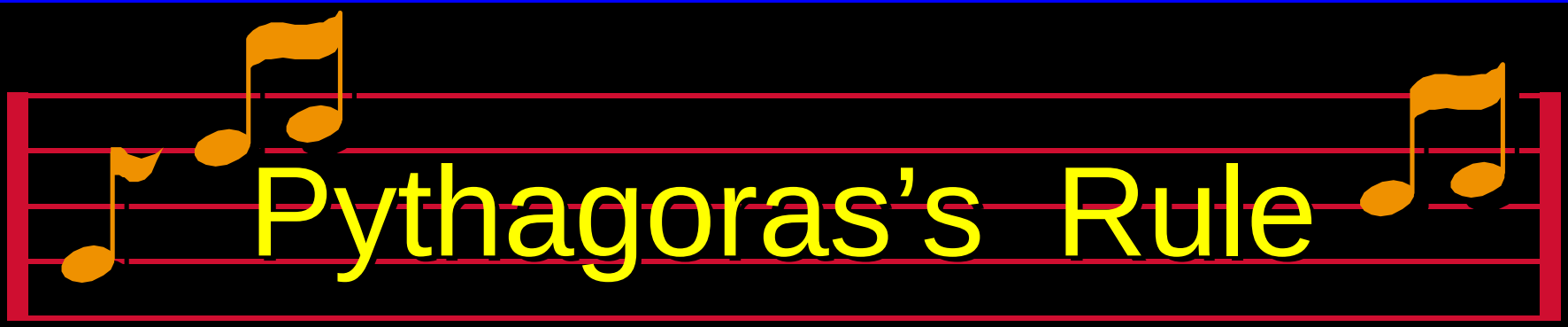
- Pythagoras (582-507 BC)
- Ratios for intervals: 1.000, 1.333, 1.500, 2.000 (unison, fourth, fifth, octave)
- Used these ratios to construct a mathematical scale.

Pythagorean Scale

- Used string lengths since frequencies were not known.
- How do you divide an octave (1.00 to 2.00) into 8 equal parts?
- Or in terms of frequencies an interval such as 220 –440 Hz?



- Multiply or divide an existing length (ratio) by 3/2 (=1.500), factor of fifths.
- If the result lies between 1 and 2, leave it as it is.



- If the answer is less than 1, double it (up an octave)
- If the answer is greater than 1, halve it (down an octave)

Pythagoras's Rule

Step #1

- Start with D_4 = 1.000 (293.7 Hz)
- Multiply D_4= 1.00 by 1.50 to get 1.5 (the fifth) which is A_4 (440 Hz).

Pythagoras's Rule

Step #2

- Start with D_4 = 1.000 (293.7 Hz)
- Divide D_4 = 1.00 by 1.50 to get 0.666 and double to get 1.333 (the fourth) which is G_4 (392 Hz).

Pythagoras's Rule

Step #3

- Start with A_4 = 1.500 (440 Hz)
- Multiply A_4 = 1.50 by 1.50 to get 2.250 and halve to get 1.125 (the major second) which is E_4 (229.6 Hz).

Pythagoras's Rule

Step #4

- Start with G_4 = 1.333 (292 Hz)
- Divide G_4 = 1.333 by 1.50 to get 0.88888 and double to get 1.777 (the minor seventh) which is C_5 (523.3 Hz).

Pentatonic Scale

- These first 5 notes D, E, G, A, C, and D again constitute the 5-note Chinese scale called pentatonic (5 tones)
- Greek scales had 7 notes called septatonic

Pythagoras's Rule

Step #5

- Start with E_4 = 1.125 (229.6 Hz)
- Multiply E_4 = 1.125 by 1.50 to get 1.6875 (the major sixth) which is B_4 (493.9 Hz).

Pythagoras's Rule

Step #6

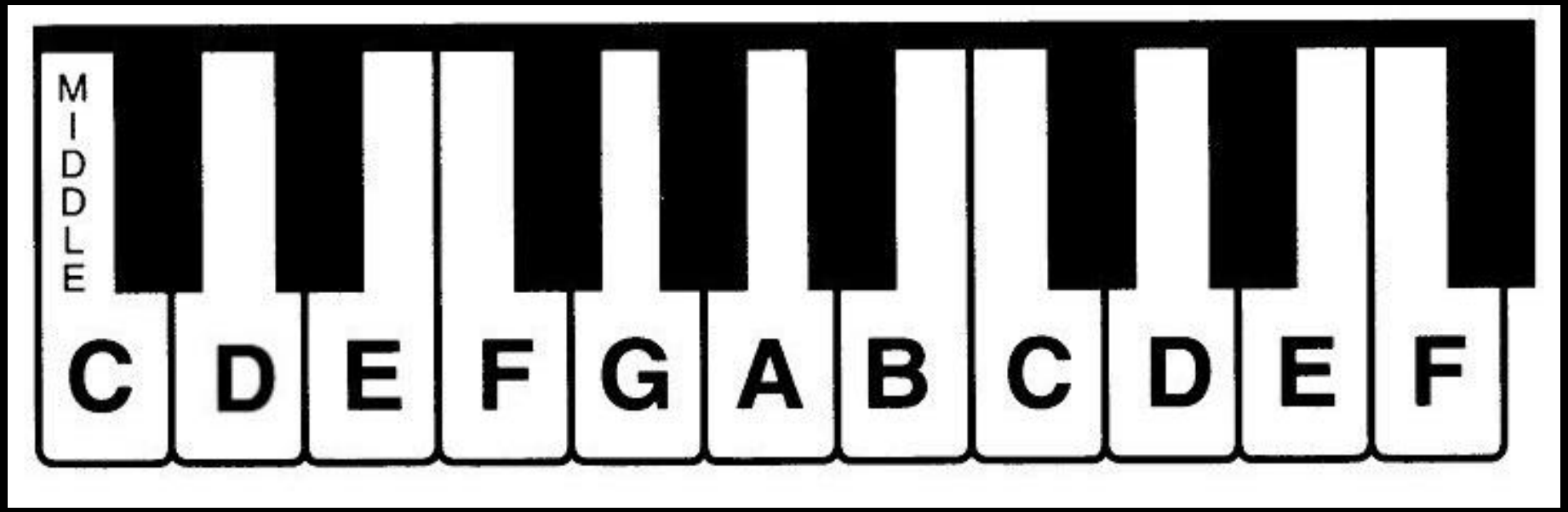
- Start with C_5 = 1.777 (523.3 Hz)
- Divide C_5 = 1.777 by 1.50 to get 1.1851 (the minor third) which is F_4 (349.2 Hz).

Pythagoras's Rule

Step #7

- Start with G_4 = 1.333 (292 Hz)
- Multiply G_4 = 1.333 by 1.50 to get 2.00 (the octave) which is D_5 (587.3 Hz).

Musical Intervals

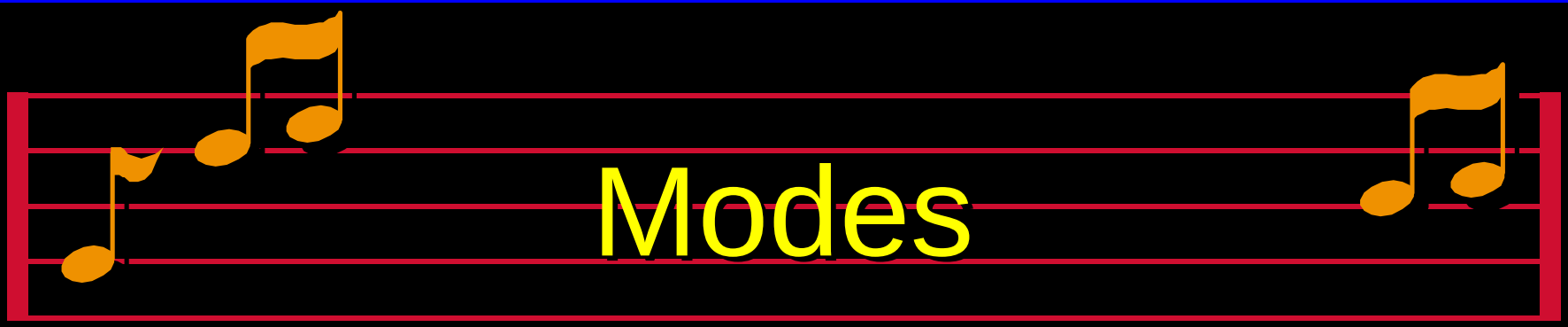


Two different ratios between adjacent notes; semitone and tone.

Tone = T and Semitone = s

T s T T T s T

D E F G A B C D



Modes

- Scales based on the white keys of the piano
- Since there are seven different named keys A, B, C, D, E, F, G, there are seven modes.

Ionian Mode or Major Scale

T T s T T T s

C D E F G A B C

Aoelian Mode or Minor Scale

T s T T s T T

A B C D E F G A

Major and Minor Scales

- Major: J.S. Bach “Well Tempered Clavier Book II” Prelude I in C major. (Track #1)
- Minor: Prelude IV in C# minor. (Track #7)

Seven Modes

- C = Ionian (major scale)
- D = Dorian
- E = Phrygian (Spanish or Oriental)
- F = Lydian (funny, comical)
- G = Mixolydian
- A = Aeolian (minor scale)
- B = Locrian (not used)

Phrygian Mode

(starts on E)

Vaughn Williams

Fantasia on a Theme by Thomas Tallis

Mathematical Basis for Scales

Multiples of 1.500 generate the same 8 note scale that was found by musicians to be the “right” ones for a musical scale.

Mathematical Basis for Scales

- The fifth is a multiple of 1.500.
- The fifth is the 3rd harmonic. ($3h_1/2h_1 = 1.50$)

Mathematical Basis for Scales

- Third harmonic of A is E.
- Third harmonic of E is B.
- Third harmonic of B is F#.
- of F# is C#, of C# is G#, of G# is D#, of D# is A#, of A# is F, of F is C, of C is G, of G is D, and of D is back to A.
- This is the entire chromatic scale!

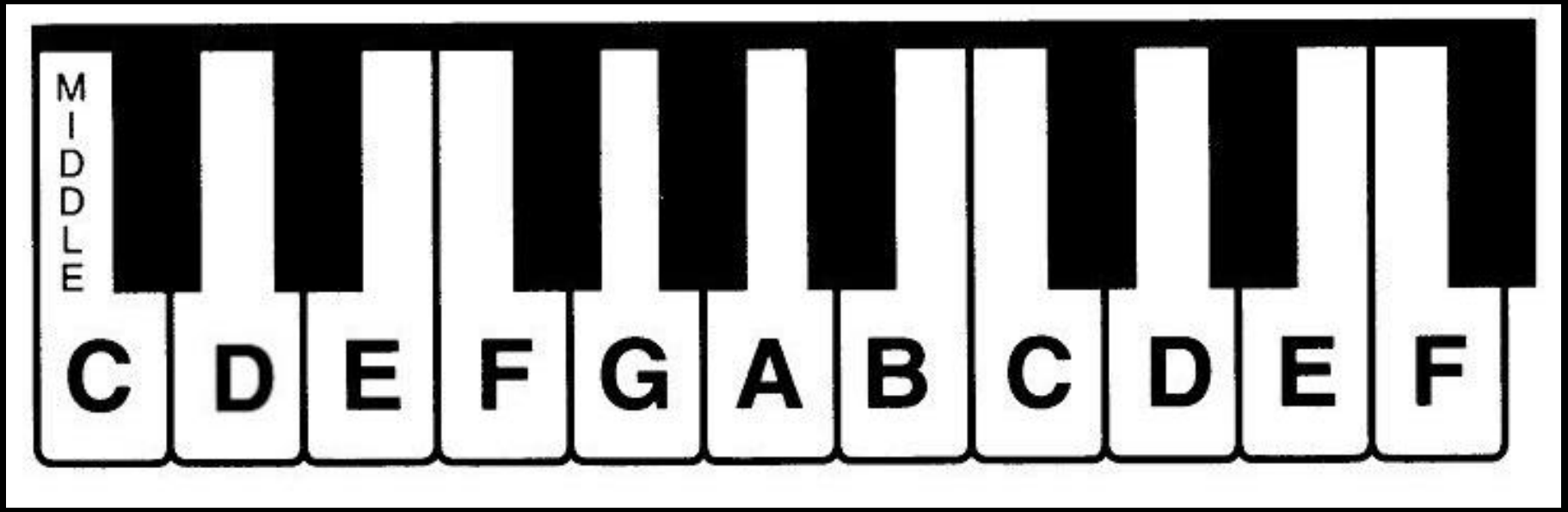
Aesthetic Basis for Scales

- The 3rd harmonic is the lowest and strongest harmonic that is not an octave.
- Stringed instruments have the 3rd harmonic.
- A scale based on 3rd harmonics should be the most “natural” or pleasing.

Equal-Tempered Scale

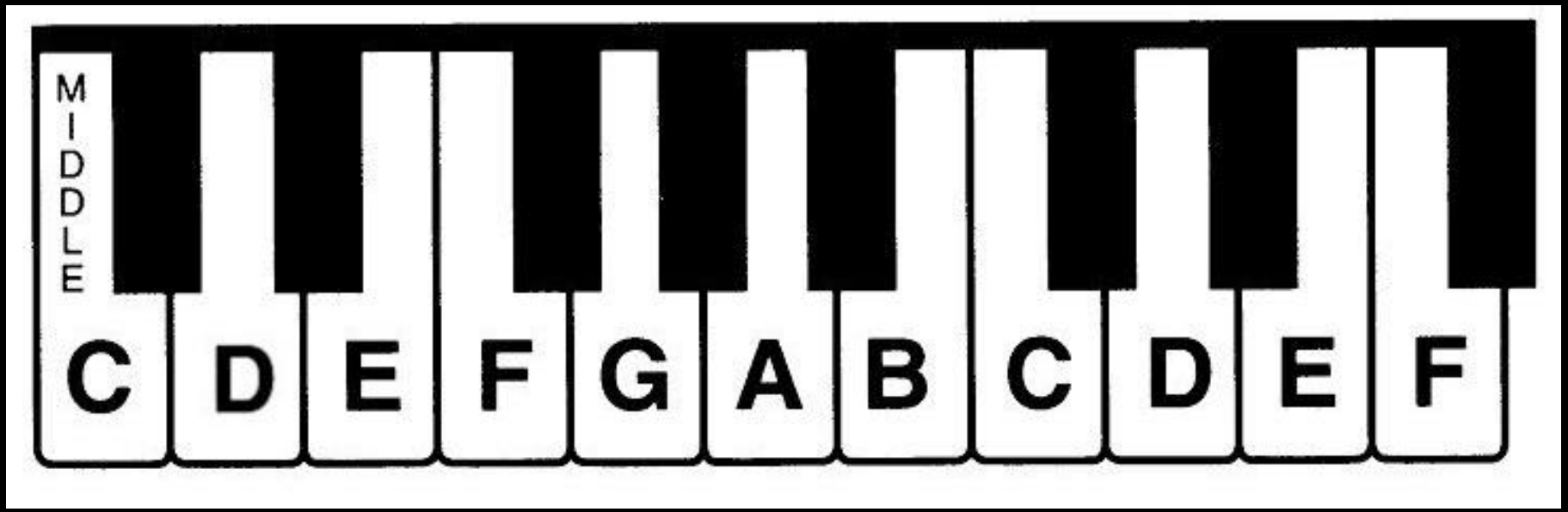
- 7 Greek modes or Church modes use all of the white keys
- Comprise 7 combinations of T=tone and s=semitone sequences
- Using a particular mode requires the scale to start on one and only one note.
- Need to place semitones anywhere.

Equal-Tempered Scale



- E-F is a semitone and so is B-C.
- Add 5 more (the black keys).

Equal-Tempered Chromatic Scale



- 12 equally spaced semitones
- $\sqrt[12]{2} \approx 1.059463...$

Major Scales

C Major

T T s T T T s

C D E F G A B C

F Major

T T s T T T s

F G A Bb C D E F

Studio - Scales.sng

Major Scales Quality

- Happy
- Strong
- Serene

Minor Scales

A Minor

T s T T s T T

A B C D E F G A

A Minor

T s T T s T T

A B C D E F G A

Studio - Scales.sng

Minor Scales Quality

- Sad
- Erie
- Troubling

Diatonic vs. Chromatic

- Diatonic Scales - major and minor scales
- Diatonic notes - notes of a particular scale
- Chromatic notes - the other notes

Equal Tempered vs. Pythagorean

- Pythagorean - the white keys determined by the rule of 3/2.
- Equal Tempered - 12 evenly spaced intervals by the factor of

$$\sqrt[12]{2} \approx 1.059463...$$

- They are similar but not the same!

Equal Tempered vs. Pythagorean

Note	Equal Tempered	Pythagorean
C	261.6	260.7
D	293.7	293.3
E	329.6	330.0
F	349.2	347.7
G	392.0	391.1
A	440.0	440.0
B	493.9	495.0